Your Roll No. ....

(12)

I

S. No. of Paper : 93

Unique Paper Code : 32351501

Name of the Paper : **Metric Spaces**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.
All questions are compulsory

1. (*a*) (i) Let $X = \mathbf{R} \cup \{\infty\} \cup \{-\infty\}$. Define the metric d on X by :

$$d(x, y) = \tan^{-1} x - \tan^{-1} y \,|, x, y \in X,$$

where $\tan^{-1}(\infty) = \pi/2$ and $\tan^{-1}(-\infty) = -\pi/2$.

Show that (X, *d*) is a metric space.

(ii) Let X denote the set of all Riemann integrable functions on $[a, b]$. For f, g in X, define:

$$d(f, g) = \int_a^b |f(x) - g(x)| \, dx.$$

Show that d is not a metric on X. 3+3=6

(*b*) Prove that a sequence in $\mathbf{R}^n$ is Cauchy in the Euclidean metric d_2 if and only if it is Cauchy in the maximum metric d_∞. 6

P. T. O.

(c) (i) Show that the metric space (X, d) of rational numbers is an incomplete metric space.

(ii) Let X be any nonempty set and d be the discrete metric defined on X. Prove that the metric space (X, d) is a complete metric space. 3+3=6

2. (a) Let (X, d) be a metric space. Prove that the intersection of any finite family of open sets in X is an open set in X. Is it true for the intersection of an arbitrary family of open sets? Justify your answer. 6

(b) Prove that if A is a subset of the metric space (X, d), then $d(A) = d(\bar{A})$. 6

(c) Let F be a subset of a metric space (X, d). Prove that the following are equivalent:

(i) $x \in \bar{F}$

(ii) $S(x, \epsilon) \cap F \neq \emptyset$ for every open ball $S(x, \epsilon)$ centered at x;

(iii) There exists an infinite sequence $\{x_n\}$, $n \geq 1$ of points (not necessarily distinct) of F such that $x_n \to x$. 6

3. (a) Let (X, d) be a metric space and $Z \subseteq Y \subseteq X$. If $cl_X(Z)$ and $cl_Y(Z)$ denote, respectively, the closures of Z in the metric spaces X and Y, then show that:

$$cl_Y(Z) = Y \cap cl_X(Z).$$ 6

(b) (i) Let Y be a nonempty subset of a metric space (X, d_X), and (Y, d_Y) is complete. Show that Y is closed in X.

(ii) Is the converse of part (i) true? Justify your answer.

4+2=6

(c) Let d_p (p ≥ 1) on the set $\mathbf{R}^n$ be given by:

$$d_p(x, Iy) = (\Sigma_{j=1}^{n}|x_j - y_j|^p)^{1/p},$$

for all $x=(x_1, x_2, \ldots, x_n)$, $y=(y_1, y_2, \ldots, y_n)$ in $\mathbf{R}^n$. Show that ($\mathbf{R}^n$, d_p) is a separable metric space.

6

4. (a) Prove that a mapping f: (X, d_X) → (Y, d_Y) is continuous on X if and only if f^{-1}(F) is closed in X for all closed subsets F of Y.

6 ½

(b) (i) Define an isometry between the metric spaces (X, d_X) and (Y, d_Y), and show that it is a homeomorphism.

(ii) Is the completeness of a metric space preserved under homeomorphism? Justify your answer.

4+2½=6½

(c) State and prove the Contraction Mapping Principle.

1½+5=6½

5. (a) Let f be a mapping of (X, d_X) into (Y, d_Y). Prove that f is continuous on X if and only if for every subset F of Y:

$$f^{-1}(F^o) \subseteq (f^{-1}(F))^o$$

6½

(b) Prove that the metrics d_1, d_2 and d_∞ defined on $\mathbf{R}^n$ by:

$$d_1(x, y) = \sum_{j=1}^{n} |x_j - y_j|;$$

$$d_2(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^2\right)^{1/2}; \text{ and}$$

$$d_\infty(x, y) = \max\{|x_j - y_j| : j = 1, 2, \ldots, n\}$$

for $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n) \in \mathbf{R}^n$ are equivalent. 6½

(c) Prove that a metric space (X, d) is disconnected if and only if there exists a continuous mapping of (X, d) onto the discrete two element space (X_0, d_0). 6½

6. (a) If every two points in a metric space X are contained in some connected subset of X, prove that X is connected. 6½

(b) Let (X, d) be a metric space and Y a subset of X. Prove that if Y is compact subset of (X, d), then Y is bounded. Is the converse true? Justify your answer. 6½

(c) If f is a one-to-one continuous mapping of a compact metric space (X, d_X) onto a metric space (Y, d_Y), then prove that f is a homeomorphism. 6½

S. No. of Question Paper : 94

(13)

Unique Paper Code : 32351502

Name of the Paper : Group Theory-II

Name of the Course : B.Sc. (H) Mathematics

Semester : V

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Question No. **1** has been divided in 10 parts

and each part is of **1½** marks.

Each question from **2** to **6** has 3 parts and each part is of

6 marks. Attempt any *two* parts from each question.

1. State true (T) or false (F). Justify your answer in brief :

(*a*) $\mathbf{Z}_2 \oplus \mathbf{Z}_3$ is isomorphic to $\mathbf{Z}_6$ where $\mathbf{Z}_n$ is used for group $\{0, 1, 2, \ldots\ldots\ldots\ldots, n-1\}$ under addition modulo n.

(*b*) The largest possible order of any element of external direct product $\mathbf{Z}_3 \oplus \mathbf{Z}_6 \oplus \mathbf{Z}_2$ is 36.

94

(c) If H, K and L are normal subgroups of a group G. Then G is internal direct product of H, K and L if G = HKL and $H \cap K \cap L = \{e\}$ where e is identity of G.

(d) The order of the group of inner automorphisms of additive group of integers is greater than 1.

(e) The dihedral group D_8 of order 8 is a subgroup of the symmetric group S_4.

(f) For any two groups G_1 and G_2, $G_1 \oplus G_2$ is isomorphic to $G_2 \oplus G_1$.

(g) Let G be a non-abelian group. A map $G \times G \to G$ is given by $(g, a) \mapsto g.a = ag$ for all g and a in G. This is an action of G on itself.

(h) Every subgroup H of a group G of index 2 is normal in G.

(i) If order of a group G is greater than 1, then the conjugacy action of G on itself is transitive.

(j) In S_3 the all conjugacy classes are $\{(1\ 2), (1\ 3), (2\ 3)\}$ and $\{(1\ 2\ 3), (1\ 3\ 2)\}$.

2. (a) Prove that for any positive integer n, $Aut(\mathbf{Z}_n)$ is isomorphic to $U(n)$, where $\mathbf{Z}_n$ is the group $\{0, 1, 2 ......, n-1\}$ under addition modulo n and $U(n)$ the group of units under multiplication modulo n and $Aut(\mathbf{Z}_n)$ denotes the group of automorphisms of $\mathbf{Z}_n$.

(b) Define the commutator subgroup G′ of a group G. Prove that G/G′ is abelian and if G/N is abelian then G′ is subgroup of N.

(c) Prove that the order of an element of a direct product of fnite number of finite groups is the least common multiple of the orders of the components of the element.

3. (a) Prove that if a group G is the internal direct product of a fnite number of subgroups $H_1, H_2, ..., H_n$, then G is isomorphic to the external direct product of $H_1, H_2, ..., H_n$.

(b) Find all subgroups of order 4 in $\mathbf{Z}_4 \oplus \mathbf{Z}_4$.

(c) Let G = {1, 7, 17, 23, 49, 55, 65, 71} be the group under multiplication modulo 96. Express G as an internal direct product of cyclic groups.

4. (a) Let G be an abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G.

(b) (i) Let G be a group acting on a non-empty set A. Define kernel of action of G on A and explain when this action will be called faithful.

(ii) Consider the action of the dihedral group D_8 of order 8 on the set $A = \{\{1, 3\}, \{2, 4\}\}$ of the unordered pair of opposite vertices of a square. Show that this action is not faithful. Further, show that for either $a \in A$ ($a = \{1, 3\}$ or $\{2, 4\}$), the stabilizer of a in D_8 equals the kernel of the action.

(c) Let G be a group and A be any subset of G. Define centralizer $C_G(A)$ and normalizer $N_G(A)$ of A in G. Further, for the symmetric group S_3 and a subgroup $A = \{I, (1, 2)\}$ of S_3, find centralizer and normalizer of A in S_3 where I denotes identity of S_3.

5. (a) Let G be a group, H be a subgroup of G and let G act by left multiplication on the set A of left cosets of H in G. Let π_H be the associated permutation representation afforded by this action. Then, show that the following hold :

(i) G acts transitively on A.

(ii) The stabilizer in G of $1H \in A$ is a subgroup of H where 1 is identity of G.

(iii) Kernel of π_H is equal to $\bigcap_{x \in G} xHx^{-1}$ and the kernel of π_H is the largest normal subgroup of G contained in H.

(b) Let G be a group acting on a non-empty set A given by $g.a$ for all $g \in G$ and for all $a \in A$. If $a, b \in A$ and $b = g.a$, for $g \in G$, then show that $G_b = gG_a g^{-1}$. Deduce that, if G acts transitively on A, then kernel of the action is $\bigcap_{g \in G} gG_a g^{-1}$ where G_x denotes stabilizer of x in G.

(c) (i) State the class equation for a finite group G. Find all conjugacy classes and their sizes in the alternating group A_4.

(ii) Let G be a group of order p^2 for some prime p. Show that it is isomorphic to either $\mathbb{Z}_{p^2}$ or $\mathbb{Z}_p \times \mathbb{Z}_p$.

6. (a) Show that for any positive integer n greater than or equal to 5, the alternating group A_n of degree n does not have a proper subgroup of index less than n.

(b) Prove that if order of a group G is 105, then it has normal Sylow 5-subgroup and normal Sylow 7-subgroup.

(c) State and prove the Index theorem. Hence or otherwise, show that there is no simple group of order 216.

[This question paper contains four printed pages.]

Sl-No: D Q.P: 1692

2018

Unique Paper Code : 235501

I

Name of the Paper : Differential Equation and Mathematical Modeling III [Paper No. 5.1]

Name of the Course : ~~Semester Mode~~ B.Sc.(Hons.) Mathematics

Semester : V

(14)

Duration: 3 Hours

Maximum Marks: 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any three parts from each questions.

1.

a) Using Laplace transform, solve the system of equations,
$$x' = x + 2y; y' = x + e^{-t},\ x(0) = y(0) = 0.$$ [6]

b) Using the factorization $s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$, show that:
$$L^{-1}\left\{\frac{s}{s^4+4a^4}\right\} = \frac{1}{2a^2}\sinh at \sin at.$$ [6]

c) Find the Frobenius series solution of
$$xy'' + 2y' + xy = 0.$$ [6]

d) Show that the function $f(t) = \sin(e^{t^2})$ is of exponential order as $t \to \infty$ but that its derivative is not. [6]

2.

a) Explain the linear congruence method for generating random numbers by giving a suitable example. Does this method have any drawbacks? Illustrate. [6]

b) Use Monte Carlo simulation to approximate the area under the curve $f(x) = \sqrt{x}$, over the interval $\frac{1}{2} \le x \le \frac{3}{2}$. [6]

c) Using simplex Method

Optimize $6x + 4y$

subject to $-x + y \le 12$

$x + y \le 24$

$2x + 5y \le 80,\ x, y \ge 0$ [6]

d) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any one time. The unloading time required for a ship depends on the type and the amount of cargo and varies from 45 to 90 minutes.

Below is given a situation with 5 ships

	ship 1	ship 2	ship 3	ship 4	ship 5
Time between successive ship(in min)	20	30	15	120	25
Unloading time	55	45	60	75	80

(i) Draw the time line diagram depicting clearly the situation for each ship, the idle time for the harbor and the waiting time. [4]

(ii) List the waiting times for all the ships and find the average waiting time. [2]

3.

a) (i) Draw the graphs of K_5, N_6, and C_7. [3]

(ii) Define a r - regular graph. Prove that, a r - regular graph with n vertices has $\frac{1}{2}nr$ edges. [3]

b) (i) Define Eulerian graph. Which one among K_8, C_8, Q_8 are Eulerian. Explain with reason. [3]

(ii) State Handshaking Lemma. [3]

c) Find two further ways of arranging fifteen dominoes from (0-0 to 4-4) in a ring. [6]

d) Prove that a connected graph is Eulerian iff each vertex has even degree. [6]

4.

a) Find two linearly independent Frobenius series solution of

$$4xy'' + 2y' + y = 0$$ [6]

b) Show that: $L\{t \cosh kt\} = \frac{s^2+k^2}{(s^2-k^2)^2}$. [6]

c) Carpenter problem is given by:
Maximize $z = 25x + 30y$
subject to $20x + 30y \leq 690$ and $5x + 4y \leq 120$ $x, y \geq 0$.
Determine the sensitivity of the optimal solution to a change in "c" using the objective function $cx + 30y$. [6]

d) Prove that if G be a graph in which every vertex has an even degree, then G can be split into cycles, such that no two cycles have an edge in common. [6]

[This Question paper contains 4 printed page(s)]

Sr. No. of Question Paper: 1693 Roll No.:..2018.........

Unique paper Code: 235503

Name of the course: B.Sc. (Hons) Mathematics

Title of the Paper: Analysis-IV(MAHT-502)

Semester: V

Duration: 3 Hours Maximum Marks: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any **TWO** parts from each question.

1. a) Let d and e be two metrics on a set X. Let g be the function defined on X x X by g(x, y) = min {e(x, y), d(x, y)}. Show that g need not be a metric on X and find a condition under which it is a metric. (6)

b) Define an isometry φ between the metric spaces (X, d) and (Y, e). Show that every isometry is injective. Further show that φ^{-1} is also an isometry. Is φ a homeomorphism? (6)

c) Suppose (X, d) is a metric space, w ε X and A is a subset of X. Then show that

$$dist(w, cl(A)) = dist(w, A),$$

where cl(A) is the closure of A. (6)

2. a) If S is a subset of a metric space X, prove that

(i) $(S^0)^c = cl(S^c)$

(ii) $S^0 = \{x \in X: dist(x, S^c) > 0\}$. (6)

b) Suppose (X, d) is a metric space and $A \subseteq X$. Show that

$$\text{diam}(\bar{A}) = \text{diam}(A)$$

Is diam (A°) = diam(A)? Justify your answer.

(6)

c) (i) Define a bounded metric space. Give an example of a bounded and an unbounded metric space.

(ii) Suppose X is a metric space and S is a bounded subset of X, Show that closure of S is bounded in X.

(3 + 3= 6)

3. a) Suppose (X, d) is a metric space and S is a nonempty subset of X. Prove that S is open in X if and only if S is a union of open balls of X.

(6)

b) Suppose X is a metric space and Z is a metric subspace of X. Suppose $r \in \mathbb{R}^+$.

If $x \in Z$, then prove that $b_X[x; r] \cap Z$ is the closed ball $b_Z[x; r]$ *of* Z. and all the closed balls of Z are of this form.

(6)

c) If A is a nonempty subset of the metric space X then show that $x \in A^{\circ}$ if, and only if, there is no sequence in A^c that converges to x in X.

(6)

4. a) (i) Let A be a nonempty subset of a metric space X. Prove that the function f: x → dist(x, A) is uniformly continuous.

(ii) Prove that a function with a discrete metric space as its domain is continuous.

(3 ½ + 3 = 6 ½)

b) Prove that a subset A of real line, **R** is totally bounded if, and only if, it is bounded.

(6 ½)

c) (i) Let (X, d) and (Y, e) are metric spaces, S is a bounded subset of X and $f : X \to Y$ is a Lipschitz function. Prove that $f(S)$ is bounded in Y.

(ii) Suppose X is a metric space and f is a strong contraction on X. Then, prove that f is uniformly continuous on its domain.

(3+3 ½ = 6 ½)

5. a) State Banach's Fixed- Point Theorem. Suppose f: **R** → **R** is a differentiable function and there exists k ε (0, 1) such that $|f'(x)| \leq k$ for all x ε **R**.

Then, show that f has a unique fixed point.

(6 ½)

b) Suppose X is a non-empty set and (Y, e) is a metric space. Show that the space B(X, Y) of bounded functions from X into Y, with its supremum metric, s, defined by

$$s(f, g) = \sup\{e(f(x), g(x)) : x \in X\} \text{ for } f, g \in B(X, Y),$$

is a complete metric space if and only if Y is complete.

(6 ½)

c) Prove that the closure of a connected subset, A, of a metric space (X, d) is connected.

(6 ½)

6. a) Suppose (X, d) is a metric space. Prove that X is connected if and only if either $X = \emptyset$ or the only continuous functions from X to the discrete two point space {0, 1} are the two constant functions.

(6 ½)

b) Suppose X is a compact metric space and $f: X \to Y$ is a continuous map. Prove that $f(X)$ is compact and that f is uniformly continuous.

(6 ½)

c) State and prove Inverse Function Theorem.

(6 ½)

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[This question paper contains 3 printed pages.] 2018

Sl. No. 2 R.P. : 1694 (16) I

Unique Paper Code : 235504

Name of the Paper : Algebra-IV (MAHT-503)

Name of the Course : B.Sc.(H) Mathematics

Semester : V

Duration : 3 hours

Maximum Marks : 75

Instruction for the candidates

1. *Write your Roll No. on the top immediately on receipt of this question paper.*
2. *Attempt any five parts from Question No.1. Each part carries three marks.*
3. *Attempt any two parts from each of the Question No. 2 to 6. Each part carries six marks.*

1. (a) Let $\beta = \{(2,1),(3,1)\}$ be an ordered basis for $\mathbb{R}^2$. Suppose that the dual basis of β is given by $\beta^* = \{f_1, f_2\}$. Then determine formulas for f_1 and f_2 explicitly.

(b) Find all the eigenvectors of the matrix $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

(c) Let A be a real $n \times n$ matrix such that $A^3 = A$. Then show that A is diagonalizable.

(d) Let $V = C([0,1])$ be the inner product space of real valued continuous functions defined on $[0,1]$, with the inner product given by

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt, \text{ for } f, g \in V.$$

For $f(t) = t$ and $g(t) = e^t$ in V, compute $\langle f, g \rangle$, $\|f\|$ and $\|g\|$.

(e) Apply the Gram-Schmidt orthogonalization process to the given subset

$$S = \{(1,0,1,0),(1,1,1,1),(0,1,2,1)\}$$

of the inner product space $V = \mathbb{R}^4$ to obtain an orthogonal basis for span(S).

(f) Let F be a field and let E be a finite extension of F. Then, prove that E is an algebraic extension of F.

(g) Find the degree and basis for $\mathbb{Q}(\sqrt{3},\sqrt{5})$ over $\mathbb{Q}$.

(h) Prove that $\sin\theta$ is constructible if and only if $\cos\theta$ is constructible.

2. (a) Let V be a finite dimensional vector space. Prove that V is isomorphic to its double dual V^{**}.

(b) Let T be a linear operator on a vector space V and let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be distinct eigenvalues of T. If $v_1, v_2, \ldots, v_k$ are eigenvectors of T such that λ_i corresponds to v_i $(1 \le i \le k)$, then $\{v_1, v_2, \ldots, v_k\}$ is linearly independent.

(c) Let T be a linear operator on a finite dimensional vector space V and let W be a T- invariant subspace of V. Then, the characteristic polynomial of T_W divides the characteristic polynomial of T.

3. (a) Let $p(t)$ be a minimal polynomial for a linear operator T on a finite dimensional vector space V and T_0 be the zero operator on V. If $g(t)$ is any polynomial for which $g(T) = T_0$, then show that $p(t)$ divides $g(t)$. Hence or otherwise, show that the minimal polynomial of T is unique.

(b) Let V be an inner product space over F. Prove that for all $x, y \in V$,

i. $\langle x,y\rangle = \frac{1}{4}\|x+y\|^2 - \frac{1}{4}\|x-y\|^2$ if $F = \mathbb{R}$;

ii. $\langle x,y\rangle = \frac{1}{4}\sum_{k=1}^{4} i^k \|x+i^k y\|^2$ if $F = \mathbb{C}$, where $i^2 = -1$.

(c) For the following set of data, use the least square approximation to find the line of best fit: $\{(-3,9),(-2,6),(0,2),(1,1)\}$.

4. (a) Define an inner product space $V(F)$, where V is a vector space over the field $F = \mathbb{R}$ or $\mathbb{C}$. Also, prove that

$$\langle x, y+z\rangle = \langle x,y\rangle + \langle x,z\rangle \text{ for } x,\ y,\ z \in V.$$

(b) Let V be an inner product space and $S = \{v_1, v_2, \ldots, v_k\}$ be an orthogonal subset of V consisting of non-zero vectors. If $y \in span(S)$, then prove that

$$y = \sum_{i=1}^{k} \frac{\langle y, v_i\rangle}{\|v_i\|^2} v_i.$$

(c) Let V be an inner product space, and let T and U be linear operators on V. Prove that

i. $(T+U)^* = T^* + U^*$

ii. $T^{**} = T$

iii. $(TU)^* = U^*T^*$,

where T^* denotes the adjoint of operator T.

5. (a) Let F be a field and let $f(x) \in F[x]$. Then prove that any two splitting fields of $f(x)$ over F are isomorphic.

(b) Show that $\mathbb{Q}(\sqrt{2},\sqrt{3}) = \mathbb{Q}(\sqrt{2}+\sqrt{3})$.

(c) If E, F, K are fields such that K is a finite extension of E and E is a finite extension of F, then prove that K is a finite extension of F and

$$[K:F] = [K:E][E:F].$$

6. (a) For each prime p and each positive integer n, prove that there is a unique finite field of order p^n up to isomorphism.

(b) Let F be a field and $f(x)$ a non-constant polynomial in $F[x]$. Prove that there is an extension field E of F in which $f(x)$ has a zero.

(c) If a and b are constructible numbers, then prove that $a+b$ and $a-b$ are also constructible.

Sl. No. of Q.P.: 1695

2018

Set A

Unique Paper Code	:	235505
Name of the Paper	:	Linear Programming and Theory of Games (MAHT 504)
Name of Course	:	B.Sc. (Hons.) Mathematics – ~~III (Three year Semester Mode)~~
Semester	:	V
Duration	:	3 Hours
Maximum Marks	:	75

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Instructions of Candidates

(i) Write your Roll No. on the top immediately on receipt of this question paper.
(ii) Attempt any two parts from each question.
(iii) All questions carry equal marks.

1. (a) Consider the LPP
Minimize $z = cx$,
subject to,

$$AX = b,$$
$$X \geq 0,$$

where A is $m \times n$ matrix with rank m. If $\hat{X}$ is an extreme point feasible solution of the feasible set, then prove that $\hat{X}$ is a basic feasible solution of the system $AX = b$, $X \geq 0$. Is the converse true?

(b) Consider the LPP
Minimize $z = cx$,
subject to,

$$AX = b,$$
$$X \geq 0.$$

Suppose there exists a basic feasible solution with $z_k - c_k > 0$ for some non-basic variable x_k and $Y_k \leq 0$, then prove that the LPP is unbounded.

(c) Solve the following LPP using simplex method:
Minimize $z = x_1 + x_2 - 4x_3$,
subject to,

$$x_1 + x_2 + 2x_3 \leq 9,$$
$$x_1 + x_2 - x_3 \leq 2,$$
$$-x_1 + x_2 + x_3 \leq 4,$$
$$x_1, x_2, x_3 \geq 0.$$

2. (a) Use two-phase simplex method to solve the following:
Maximize $z = 2x_1 + 3x_2$,
subject to

$$3x_1 + 3x_2 \geq 4,$$
$$x_1 + x_2 \leq 1,$$
$$x_1 \geq 0,\ x_2 \text{ unrestricted}.$$

(b) Use Big-M method to solve the following:
Minimize $z = -3x_1 + 2x_2$,
subject to

$$x_1 + x_2 \le 1,$$
$$x_1 + x_2 \ge 1,$$
$$x_1,\ x_2 \ge 0.$$

(c) Find the inverse of the matrix $\begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$ using simplex method.

3. (a) Find the dual of the following problem:
Minimize $z = x_1 + x_2 + 2x_3$,
subject to

$$x_1 + 2x_2 \ge 3,$$
$$x_2 + 7x_3 \le 6,$$
$$x_1 - 3x_2 + 5x_3 = 5,$$
$$x_1,\ x_2 \ge 0,$$
x_3 unrestricted.

(b) If x is a feasible solution of
Maximize $z = cx$,
subject to

$$Ax \le b,$$
$$x \ge 0,$$

and w is any dual feasible solution of
Minimize $z^* = b^T w$,
subject to

$$A^T w \ge c^T,$$
$$w \ge 0,$$

then show that $cx \le b^T w$.

(c) Solve the following L.P.P. by solving its dual:
Maximize $z = -2x_1 - 3x_2$,
subject to

$$2x_1 - x_2 \le 3,$$
$$x_2 \ge 4,$$
$$x_1 + 3x_2 \ge 10,$$
$$4x_1 - x_2 \le -10,$$
$x_1 \ge 0$, x_2 unrestricted.

4. (a) Solve the following cost-minimizing transportation problem. Use Vogel's approximation method to find initial basic feasible solution:

	D_1	D_2	D_3	Supply
O_1	6	8	4	14
O_2	4	9	3	12
O_3	1	2	6	5
Demand	6	10	15	

(b) Solve the following cost-minimizing assignment problem:

	a	b	c	d	e
A	3	3	1	5	1
B	7	5	1	11	3
C	1	2	1	2	3
D	3	5	1	5	3
E	3	1	1	1	3

(c) Use the minimax criteria to find best strategy for each player, saddle point and value of the game having following pay off matrix:

$$\text{Player A}\ \overset{\text{Player B}}{\begin{bmatrix} -3 & -2 & 6 \\ 2 & 0 & 2 \\ 5 & -2 & -4 \end{bmatrix}}.$$

Is the game fair? Give reason(s).

5. (a) Use the relation of dominance to solve the game whose pay-off matrix is given by

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 5 \\ 0 & 1 & -1 \end{bmatrix}$$

and hence find the optimum strategies and value of the game.

(b) Solve graphically the game whose pay off matrix is

$$\begin{bmatrix} 4 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

(c) Reduce the following game to a Linear Programming Problem and then solve by simplex method:

$$\begin{bmatrix} 1 & -4 \\ -3 & 4 \\ 2 & 2 \end{bmatrix}.$$

Sl-No of Q.P : 1892 7/12 2018

F-11

Unique Paper Code	:	2351503
Name of the Paper	:	Calculus II (Multivariate Calculus)
Name of the Course	:	B.Sc.(H) Mathematics
Semester	:	V
Duration	:	3 hours
Maximum Marks	:	75

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Instructions for Candidates

(i) Attempt any five questions from each section.

(ii) Each question carries 5 marks.

(iii) Use of scientific calculators is allowed.

Section-I

1. Examine the following functions for continuity at origin:

a) $f(x,y) = \frac{2xy}{x^2+y^2}$

b) $f(x,y) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

2. When two resistances R_1 and R_2 are connected in parallel, the total resistance R satisfies $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 is measured as *300 ohms* with a maximum error of 2 percent and R_2 is measured as *500* ohms with a maximum of *3* percent, what is the maximum percentage error in R?

3. If y is a differentiable function of x such that $\sin(x + y) + \cos(x - y) = y$, find $\frac{dy}{dx}$.

4. Let T(x, y) be the temperature at each point (x, y) in a portion of the plane that contains the ellipse $x = 2\cos(t)$, $y = \sin(t)$ for $0 \leq t \leq 2\pi$. Suppose $\frac{\partial T}{\partial x} = y$ and $\frac{\partial T}{\partial y} =$ [illegible] → x

a) Find $\frac{dT}{dt}$ and $\frac{d^2T}{dt^2}$

b) Locate the maximum and minimum temperatures on the ellipse.

5. Find the equations of the tangent plane and normal line at the point $P_0(1,-1,2)$ on the surface S given by $x^2y + y^2z + z^2x = 5$.

6. Find the absolute extrema of the function $f(x, y) = e^{x^2-y^2}$ over the disc $x^2 + y^2 \leq 1$.

Section-II

7. A container in $\mathbb{R}^3$ has the shape of the cube given by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$. A plate is placed in the container in such a way that it occupies that portion of the plane $x + y + z = 1$ that lies in the cubical container. If the container is heated so that the temperature at each point (x, y, z) is given by $T(x, y, z) = 4 - 2x^2 - y^2 - z^2$ in hundreds of degrees Celsius. What are the hottest and coldest points on the plate?

8. Find the area of the region between y = cos(x) and y = sin(x) over the interval $0 \le x \le \frac{\pi}{4}$ using
   a) Single Integral
   b) A double integral.
9. Show that volume of the sphere of radius R is $\frac{4}{3}\pi R^3$.
10. Evaluate $\iiint_B z^2 y e^x \, dV$, where B is the box given by $0 \le x \le 1, 1 \le y \le 2, -1 \le z \le 1$.
11. Evaluate $\iiint_D x \, dV$, where D is the solid in the first octant bounded by the cylinder $x^2 + y^2 = 4$ and the plane $2y + z = 4$.

12. Find the centroid of the Solid with constant density δ and bounded below by the xy plane on the sides by the cylinder $x^2 + y^2 = 4$ and above by the surface $z = x^2 + y^2$.

Section III

13. Suppose that the vector field F and curl F are both continuous in a simply connected region D of $\mathbb{R}^3$. Show that F is conservative in D if and only if Curl F = 0.
14. Show that no work is performed when an object moves along a closed path in a connected domain where the force field is conservative.
15. Evaluate the line integral $\int_C xy\,ds$ where C consists of the line segment C_1 from (-3, 3) to (0, 0) followed by the portion of the curve C_2: $16y = x^4$ between (0, 0) and (2, 1).

16. Find the mass of a lamina of density $\delta(x, y, z) = z$ in the shape of the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$.

17. Evaluate $\iint (x + y + z)\,dS$ where S is the surface determined by $R(u, v) = ui + uj - vk$, $0 \le u \le 1, 0 \le v \le 2$.
18. State divergence theorem and use it to evaluate $\iint_S F.N\,dS$ where $F = xyi - z^2k$ and S is the surface of the upper five faces of the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$.